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EDGE-TO-EDGE DETOUR DISTANCE IN GRAPHS

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Abstract

In this paper, we introduce the edge-to-edge $e - e'$ path, the edge-to-edge detour distance $D(e, e')$, the edge-to- edge $e - v$ detour, the edge-to-edge detour eccentricity $e_{D3}(e)$, the edge-to-edge detour radius R_3 , and the edge-to-edge detour diameter D_3 of a connected graph G, where e, e′ are edges in G. We determine these parameters for some standard graphs. It is shown that $R_3 \leq D_3 \leq 2R_3+1$ for every connected graph G and that every two positive integers a and b with $a \leq b \leq 2a+1$ are realizable as the edge-to-edge detour radius and the edge-to-edge detour diameter, respectively, of some connected graph. Also it is shown that for any two positive integers a, b with $a \leq b$ are realizable as the edge-to-edge radius and the edge-to-edge detour radius, respectively, of some connected graph and also for any two positive integers a, b with $a \leq b$ are realizable as the edge-to-edge diameter and the edge-to-edge detour diameter, respectively, of some connected graph. Also we introduce the edge-to-edge detour center $C_{D3}(G)$ and the edge-to-edge detour periphery $P_{D3}(G)$. It is shown that the edge-to-edge detour center of every connected graph does not lie in a single block of G.

Key words : distance, detour distance, edge-to-edge detour distance.
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1. Introduction

By a graph G = (V, E) we mean a finite undirected connected simple graph. For basic graph theoretic terminologies, we refer to Chartrand and Zhang [4]. If X ⊆ V, then X is the subgraph induced by X. For example if one is locating an emergency facility like police station, fire station, hospital, school, college, library, ambulance depot, emergency care center, etc., then the primary aim is to minimize the distance between the facility and the location of a possible emergency.

In 1964, Hakimi [6] considered the facility location problems as vertex-to-vertex distance in graphs. For any two vertices u and v in a connected graph G, the distance d(u, v) is the length of a shortest u − v path in G. For a vertex v in G, the eccentricity e(v) of v is the distance between v and a vertex farthest from v in G. The minimum eccentricity among the vertices of G is its radius and the maximum eccentricity is its diameter, denoted by rad(G) and diam(G) respectively. A vertex v in G is a central vertex if e(v) = rad(G) and the subgraph induced by the central vertices of G is the center Cen(G) of G. A vertex v in G is a peripheral vertex if e(v) = diam(G) and the subgraph induced by the peripheral vertices of G is the periphery Per(G) of G. If every vertex of G is a central vertex then G is called self-centered graph.

For example if one is making an election canvass or circular bus service the distance from the location is to be maximized. In 2005, Chartrand et. al. [3] introduced and studied the concepts of detour distance in graphs. For any two vertices u and v in a connected graph G, the detour distance D(u, v) is the length of a longest $u - v$ path in G. For a vertex v in G, the detour eccentricity $e_D(v)$ of v is the detour distance between v and a vertex farthest from v in G. The minimum detour eccentricity among the vertices of G is its detour radius and the maximum detour eccentricity is its detour diameter, denoted by $rad_D(G)$ and $diam_D(G)$ respectively. The detour center, the detour self-centered and the detour periphery of a graph are defined similar to the center, the self-centered and the periphery of a graph, respectively.

For example when a railway line, pipe line or highway is constructed, the distance between the respective structure and each of the communities to be served is to be minimized. In a social network an edge represents two individuals having a common interest. Thus the centrality with respect to edges have interesting applications in social networks. In 2010, Santhakumaran [8] introduced the facility locational problem as edge-to-edge distance in graphs as follows: For any edges e and e′ in a connected graph G, the edge-to-edge distance is defined by $d(e, e') = \min\{d(u, v) : u \in e, v \in e'\}$. The edge-to-edge eccentricity of e is defined by $e_3(e) = \max\{d(e, e') : e' \in E\}$. An edge f of G such that $e_3(e) = d(e, f)$ is called an edge-to-edge eccentric edge of e. The edge-to-edge radius r_3 of G is defined by $r_3 = \min\{e_3 (e) : e \in E\}$ and the edge-to-edge diameter d_3 of G is defined by $d_3 = \max\{e_3 (e) : e \in E\}$. An edge e for which e_3 (e) is minimum is called an edge-to-edge central edge of G and the set of all edge-to-edge central edges of G is the edge-to-edge center $C_3(G)$ of G. An edge e for which $e_3(e)$ is maximum is called an edge-to-edge peripheral edge of G and the set of all edge-to-edge peripheral edges of G is the edge-to-edge periphery $P_3(G)$ of G. If every edge of G is an edge-to-edge central edge then G is called the edge-to-edge self-centered graph.

These motivated us to introduce a distance called the edge-to-edge deotur distance in graphs and investigate certain results related to edge-to-edge detour distance and other distances in graphs. These ideas have interesting applications in channel assignment problem in radio technologies. Also there are useful applications of these concepts to security based communication network design. Throughout this paper, G denotes a connected graph with at least two vertices.

2. Edge-To-Edge Detour Distance

Definition 2.1. Let e and e′ be any two edges in a connected graph G. An edge-to-edge $e - e'$ path P is a $u - v$ path, where $u \in e$ and $v \in e'$ such that P contains no vertices of e and e′ other than u and v respectively. The edge-to-edge detour distance $D(e, e')$ is the length of a longest $e - e'$ path in G. An $e - e'$ path of length $D(e, e')$ is called an edge-to-edge $e - e'$ detour or simply $e - e'$ detour. For our convenience an $e - e'$ path of length $d(e, e')$ is called an edge-to-edge $e - e'$ geodesic or simply $e - e'$ geodesic.

Example 2.2. Consider the graph G given in Fig 2.1. For the edges $e = \{u, w\}$ and $e' = \{r, v\}$ in G, the paths P_1 : w, v; P_2 : u, z, r; P_3 : u, t, s, x, z, r and P_4 : u, t, s, x, y, z, r are $e - e'$ paths, while the paths Q_1 : u, w, v and Q_2 : w, u, z, r, v are not $e - e'$ paths. Now the edge-to-edge distance $d(e, e') = 1$ and the edge-to-edge detour distance $D(e, e') = 6$.

Also P_1 is an $e - e'$ geodesic and P_4 is an $e - e'$ detour.

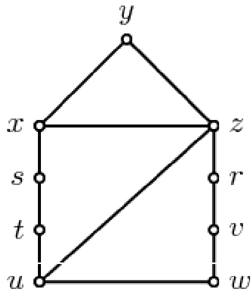

Fig 2.1: G

Since the length of an $e - e'$ path between any two edges e and e′ in a graph G of order n is at most $n - 2$, we have the following theorem.

Theorem 2.3. For any two edges e and e′ in a non-trivial connected graph G of order n, $0 \leq d(e, e') \leq D(e, e') \leq n - 2$.

Remark 2.4. The bounds in the Theorem 2.3 are sharp. For any two adjacent edges in a path of order n, $d(e, e') = D(e, e') = 0$ and for any two adjacent edges in a cycle of order n, $d(e, e') < D(e, e') = n-2$. If G is a tree of order n then, $d(e, e') = D(e, e') < n-2$.

Theorem 2.5. Let $K_{n,m}$ $(n < m)$ be a complete bipartite graph with the partition V_1, V_2 of $V(K_{n,m})$ such that $|V_1| = n$ and $|V_2| = m$. Let e and e′ be any two edges in $K_{n,m}$, then $D(e, e') = 2n-2$.

Definition 2.6. The edge-to-edge detour eccentricity $e_{D3}(e)$ of an edge e in a connected graph G is defined as $e_{D3}(e) = \max \{D(e, e') : e' \in E\}$. An edge f of G such that $e_{D3}(e) = D(e, f)$ is called an edge-to-edge detour eccentric edge of e. The edge-to-edge detour radius of G is defined as, $R_3 = rad_{D3}(G) = \min\{e_{D3}(e) : e \in E\}$ and the edge-to-edge detour diameter of G is defined as, $D_3 = diam_{D3}(G) = \max\{e_{D3}(e) : e \in E\}$. An edge e in G is called an edge-to-edge detour central edge if $e_{D3}(e) = R_3$ and the edge-to-edge detour center of G is defined as, $C_{D3}(G) = Cen_{D3}(G) = \{e \in E : e_{D3}(e) = R_3\}$. An edge e in G is called an edge-to-edge detour peripheral edge if $e_{D3}(e) = D_3$ and the edge-to-edge detour periphery of G is defined as, $P_{D3}(G) = Per_{D3}(G) = \{e \in E : e_{D3}(e) = D_3\}$. If every edge of G is an edge-to-edge detour central edge, then G is called an edge-to-edge detour self centered graph. If G is the edge-to-edge detour self centered graph then G is called the edge-to-edge detour periphery.

Example 2.7. For the connected graph G given in Fig. 2.2, the set of all edges in G are given by, $E = \{e_1 = \{v_1, v_2\}, e_2 = \{v_1, v_3\}, e_3 = \{v_2, v_3\}, e_4 = \{v_3, v_4\}, e_5 = \{v_2, v_4\}, e_6 = \{v_4, v_5\}\}$.

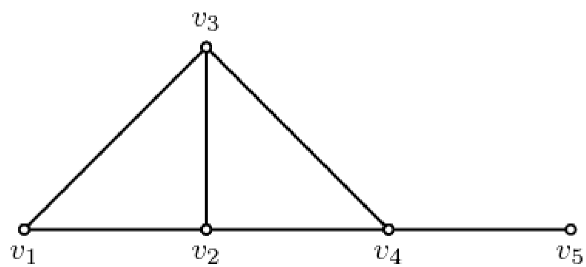

Fig. 2.2: G

The edge-to-edge eccentricity $e_3(e)$, the edge-to-edge detour eccentricity $e_{D3}(e)$ of all the edges of G are given in Table 1.

e	e_1	e_2	e_3	e_4	e_5	e_6
$e_3(e)$	1	1	1	1	1	1
$e_{D_3}(e)$	2	2	1	2	2	2

Table 1

The edge-to-edge detour eccentric edge of all the edges of G are given in Table 2.

Edge e	*Edge-to-Edge Detour Eccentric Edge*
e_2, e_3, e_5, e_6	e_1
e_1, e_3, e_4, e_6	e_2
e_3, e_5	e_4
e_3, e_4	e_5
e_1, e_2, e_3	e_6

Table 2

The edge-to-edge radius r_3 = 1, the edge-to-edge diameter d_3 = 1, the edge-to-edge detour radius R_3 = 1 and the edge-to-edge detour diameter D_3 = 2. Also the edge-to-edge center $C_3(G)$ ={e_1, e_2, e_3, e_4, e_5}, the edge-to-edge periphery $P_3(G)$ = {e_1, e_2, e_3, e_4, e_5}, the edge-to-edge detour center $C_{D3}(G)$ = {e_3} and the edge-to-edge detour periphery $P_{D3}(G)$ = {e_1, e_2, e_4, e_5}. It shows that edge-to-edge self-centered graph need not be edge-to-edge detour self-centered graph.

The edge-to-edge detour radius R_3 and the edge-to-edge detour diameter D_3 of some standard graphs are given in Table 3.

G	K_n	P_n	$C_n(n \geq 4)$	$W_n(n \geq 4)$	$K_{n,m}(m \geq n)$
R_3	$n-2$	$\lfloor \frac{n-3}{2} \rfloor$	$n-2$	$n-2$	$2n-2$
D_3	$n-2$	$n-3$	$n-2$	$n-2$	$2n-2$

Table 3

Example 2.8. The complete graph K_n, the cycle C_n, the wheel W_n and the complete bipartite graph $K_{n,m}$ are the edge-to-edge detour self centered graphs.

The following theorem is a consequence of Theorem 2.3.

Theorem 2.9. Let G be a connected graph. Then

(i) $0 \le e_3(e) \le e_{D3}(e) \le n - 2$ for every edge e in G.

(i) $0 \le r_3 \le R_3 \le n - 2$.

(ii) $0 \le d_3 \le D_3 \le n - 2$.

Remark 2.10. The bounds in the Theorem 2.9 (i) are sharp. If $G = P_3$, then $e_3(e) = e_{D3}(e) = 0$ for every edge e in G and if $G=C_n$, then $e_3(e) = e_{D3}(e) = n - 2$ for every edge e in G. Also we note that if G is a tree, then $e_3(e) = e_{D3}(e)$ for every edge e in G and for the graph G given in Fig. 2.1, $0 < e_3(e) < e_{D3}(e) < n - 2$, where $e =\{u, z\}$.

Theorem 2.11. For every connected graph G, $R_3 \le D_3 \le 2R_3+1$.

Proof. By definition $R_3 \le D_3$. Now let $P : u_1, u_2, \ldots , u_{n-1}, u_n$ be an edge-to-edge diametral path of length D_3 connecting an edge e and e′, where $e =\{u_1, u_2\}$ and $e' =\{u_{n-1}, u_n\}$, so that $D_3 = D(e, e') = D(u_2, u_{n-1})$ and let f be a edge of G such that $e_{D3}(f) = R_3 = D(y, u_{n-1}) = D(x, u_2)$, where $f = \{x, y\}$. It follows that $D_3 = D(e, e') \le D(e, f)+D(x, y)+D(f, e') \le R_3+1+R_3 \le 2R_3 + 1$.

Remark 2.12. *The bounds in the Theorem 2.11 are sharp. For the graph* G *given in Fig 2.3, it is easy to verify that* $R_3 = 2$ *and* $D_3 = 5$.

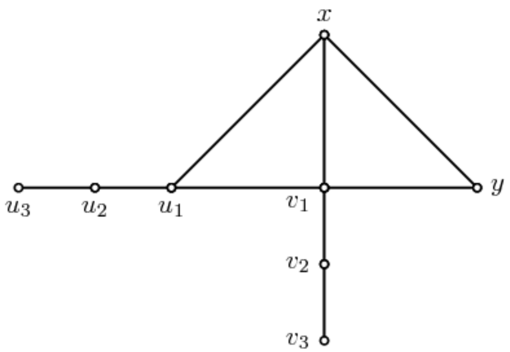

Fig. 2.3: G

Ostrand [7] showed that every two positive integers a and b with $a \leq b \leq 2a$ are realizable as the radius and diameter respectively of some connected graph and Chartrand et. al. [3] showed that every two positive integers a and b with $a \leq b \leq 2a$ are realizable as the detour radius and detour diameter respectively of some connected graph. Now we have a realization theorem for the edge-to-edge detour radius and the edge-to-edge detour diameter of some connected graph.

Theorem 2.13. For each pair a, b of positive integers with $a \leq b \leq 2a+1$, there exists a connected graph G with $R_3 = a$ and $D_3 = b$.

Proof. Case 1. $a = b$. Let $G = C_{a+3}$: $u_1, u_2, \ldots, u_{a+3}, u_1$ be a cycle of order $a + 3$. Then $e_{D3}(u_i u_{i+1}) = a$ for $1 \leq i \leq a + 2$. Thus $R_3 = a$ and $D_3 = b$ as $a = b$.

Case 2. $b \leq 2a$. Let C_{a+3} : $u_1, u_2, \ldots, u_{a+3}, u_1$ be a cycle of order $a + 3$

and $P_{b-a+1} : v_1, v_2,\dots, v_{b-a+1}$ be a path of order $b - a + 1$. We construct the graph G of order $b + 3$ by identifying the vertex u_1 of C_{a+3} and v_1 of P_{b-a+1} as shown in Fig. 2.4. It is easy to verify that $e_{D3}(u_1 u_2) = e_{D3}(u_1 u_{a+3}) = a$. Also $e_{D3}(u_i u_{i+1}) = b-i+2$ for $2 \le i \le \lceil (a+3)/2 \rceil$ and $e_{D3}(u_iu_{i+1}) = b-a+i-2$ for $\lceil (a+3)/2 \rceil < i \le a+2$. Also $e_{D3}(v_iv_{i+1}) = a+i$ for $1 \le i \le b - a$. In particular, $e_{D3}(u_2 u_3) = e_{D3}(u_{a+2} u_{a+3}) = e_{D3}(v_{b-a} v_{b-a+1}) = b$. It is easy to verify that there is no edge e in G with $e_{D2}(e) < a$ and there is no edge e′ in G with $e_{D2} (e′) > b$. Thus $R_2 = a$ and $D_2 = b$ as $a < b \le 2a$.

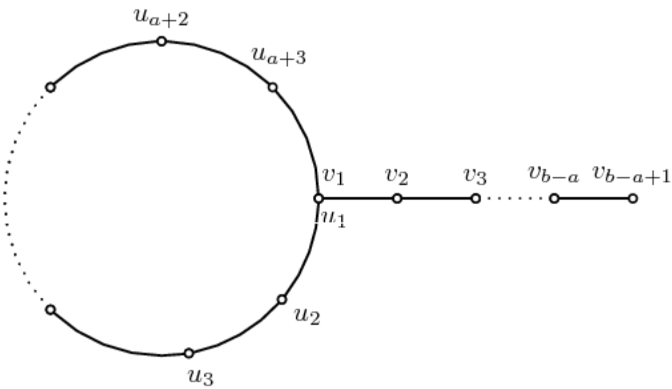

Fig. 2.4: G

Case 3. $b = 2a + 1$. Construct the graph G as shown in Fig 2.5, it is easy to verify that $e_{D3}(xv_1) = a$ and $e_{D3}(v_{b-a}v_{b-a+1}) = b$. Also there is no edge e in G with $e_{D3}(e) < a$ and there is no edge e′ in G with $e_{D3}(e′) > b$. Thus $R_3=a$ and $D_3 = b$ as $b =2a+1$.

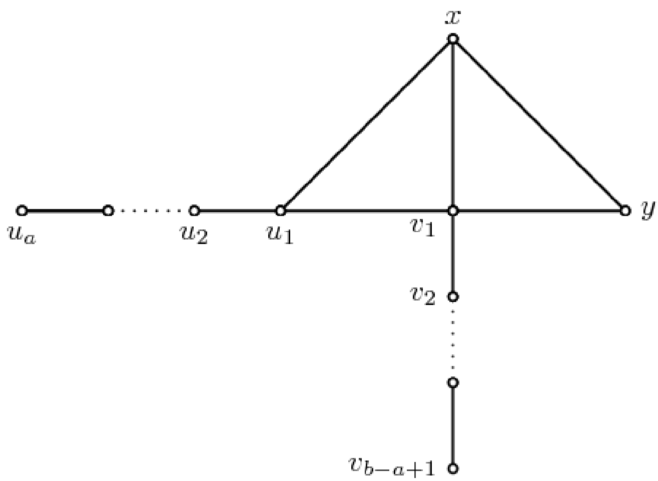

Fig. 2.5: G

Chartrand et. al. [3] showed that every pair a, b of positive integers with a ≤ b is realizable as the radius and the detour radius of some connected graph. Now we have a realization theorem for the edge-to-edge radius and the edge-to-edge detour radius of some connected graph.

Theorem 2.14. For each pair a, b of positive integers with a ≤ b, there exists a connected graph G such that $r_3 = a$ and $R_3 = b$.

Proof. Case 1. a = b. Let $P_1 : u_1, u_2, \ldots, , u_a, u_{a+2}$ and $P_2 : v_1, v_2, \ldots, v_a, v_{a+2}$ be two paths of order a+2. We construct the graph G of order 2a + 4 by joining u_1 in P_1 and v_1 in P_2 by an edge. Then $e_3(u_1 v_1) = e_{D3}(u_1 v_1) = a$ and $e_3(u_i u_{i+1}) = e_3(v_i v_{i+1}) = a + i$ for $1 \leq i \leq a+1$. It is easy to verify that there is no edge e in G with $e_3(e) = e_{D3}(e) < a$. Thus $r_3 = a$ and $R_3 = b$ as a = b.

Case 2. $a < b$. We have the following two subcases:

Subcase 1 of Case 2. $a = 1$. Any complete graph of order K_{b+2} is the desired graph.

Subcase 2 of Case 2. $a \geq 2$. Let $P_1 : u_1, u_2, \ldots, u_a, u_{a+2}$ and $Q_1 : v_1, v_2, \ldots, v_a, v_{a+2}$ be two paths of order $a + 2$. Let $P_2 : w_1, w_2, \ldots, w_{b-a+2}$ and $Q_2 : z_1, z_2, \ldots, z_{b-a+2}$ be two paths of order $b - a + 2$. We construct the graph G of order $2b + 2$ as follows: (i) identify the vertices u_1 in P_1 with w_1 in P_2 and also identify the vertices v_1 in Q_1 with z_1 in Q_2 (ii) identify the vertices u_3 in P_1 with w_{b-a+2} in P2 and also identify the vertices z_{b-a+2} in Q_2 with v_3 in Q_1 (iii) join each vertex w_i $(2 \leq i \leq b - a + 1)$ in P_2 with u_2 in P_1 and join each vertex z_i $(2 \leq i \leq b - a + 1)$ in Q_2 with v_2 in Q_1 (iv) join u_1 in P_1 with v_1 in Q_1. The resulting graph G is shown in Fig. 2.6.

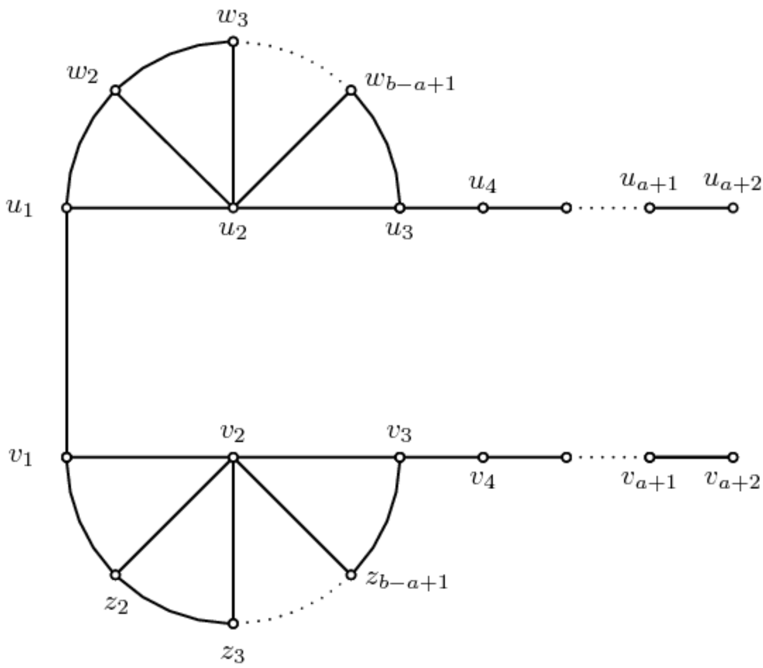

Fig. 2.6: G

It is easy to verify that

$$e_3(u_1v_1) = a$$
$$e_3(u_iu_{i+1}) = a + i \quad \text{if} \quad 1 \leq i \leq a + 1$$
$$e_3(v_iv_{i+1}) = a + i \quad \text{if} \quad 1 \leq i \leq a + 1$$
$$e_3(w_iw_{i+1}) = \begin{cases} a+1, & \text{if} \quad i = 1 \\ a+2, & \text{if} \quad i = 2 \\ a+3, & \text{if} \quad 3 \leq i \leq b - a + 1 \end{cases}$$

$$e_3(z_i z_{i+1}) = \begin{cases} a+1, & \text{if } i=1 \\ a+2, & \text{if } i=2 \\ a+3, & \text{if } 3 \le i \le b-a+1 \end{cases}$$

$e_3(u_2 w_i) = a+2$ if $1 \le i \le b-a+1$

$e_3(v_2 z_i) = a+2$ if $1 \le i \le b-a+1$

$e_{D_3}(u_1 v_1) = b$

$$e_{D_3}(u_i u_{i+1}) = \begin{cases} b+1, & \text{if } i=1 \\ 2b-a+i, & \text{if } 2 \le i \le a+1 \end{cases}$$

$$e_{D_3}(v_i v_{i+1}) = \begin{cases} b+1, & \text{if } i=1 \\ 2b-a+i, & \text{if } 2 \le i \le a+1 \end{cases}$$

$$e_{D_3}(w_i w_{i+1}) = \begin{cases} b+1, & \text{if } i=1 \\ 2b-a+2, & \text{if } 2 \le i \le b-a+1 \end{cases}$$

$$e_{D_3}(z_i z_{i+1}) = \begin{cases} b+1, & \text{if } i=1 \\ 2b-a+2, & \text{if } 2 \le i \le b-a+1 \end{cases}$$

$e_{D_3}(u_2 w_i) = b+i$ if $1 \le i \le b-a+1$

$e_{D_3}(v_2 z_i) = b+i$ if $1 \le i \le b-a+1$

It is easy to verify that there is no edge e in G with $e_3(e) < a$ and $e_{D3}(e) < b$. Thus $r_3 = a$ and $R_3 = b$ as $a < b$.

Chartrand et. al. [3] showed that every pair a, b of positive integers with $a \le b$ is realizable as the diameter and the detour diameter of some connected graph. Now we have a realization theorem for the edge-to-edge diameter and the edge-to-edge detour diameter of some connected graph.

Theorem 2.15. For any two positive integers a, b with $a \le b$, there exists a connected graph G such that $d_3 = a$ and $D_3 = b$.

Proof. Case 1. $a = b$. Let $P_{a+2}: u_1, u_2, \ldots, , u_{a+2}, u_{a+3}$ be a path of order a + 3. Then $e_3(u_i\, u_{i+1}) = e_{D3}(u_i\, u_{i+1}) = a - i + 2$ for $1 \le i \le \lceil (a+2)/2 \rceil$ and

$e_3(u_i\ u_{i+1}) = e_{D3}\ (u_i\ u_{i+1}) = i - 2$ for $\lceil (a+2)/2 \rceil < i \le a + 1$. In particular $e_3(u_1\ u_2) = e_{D3}(u_1\ u_2) = e_3\ (u_{a+2}\ u_{a+3}) = e_{D3}(u_{a+2}\ u_{a+3}) = a$. It is easy to verify that there is no edge e in G with $e_3(e) = e_{D3}(e) > a$. Thus $d_3= a$ and $D_3= b$ as a = b.

Case 2. a < b. We have the following two subcases:

Subcase 1of Case 2. a = 1. Any complete graph of order K_{b+2} is the desired graph.

Subcase 2 of Case 2. a = 2. Let G be the graph obtained by joining any one vertex of the complete graph K_b of order b with any vertex of a path $P_3 : x_1, x_2, x_3, x_4$ of order 4. It is easy to verify that $e_3(x_3x_4) = a$ and $e_{D3}(x_3x_4) = b$. Also there is no edge e in G with $e_3(e) > a$ and $e_{D3}(e) > b$.

Subcase 3 of Case 2. $a \ge 3$. Let $P_1 : u_1, u_2 , \ldots , u_{a+1} , u_{a+2}$ be a path of order a + 1. Let $P_2 : w_1, w_2 , \ldots , w_{b-a+2}$ be a path of order b − a + 2. Let $P_3 : x_1 , x_2$ be a path of order 2. We construct the graph G of order b + 3 as follows: (i) identify the vertices u_1 in P_1, w_1 in P_2 with x_1 in P_3 and identify the vertices u_3 in P_1 with w_{b-a+2} in P_2 (ii) join each vertex w_i ($2 \le i \le b - a + 1$) in P_2 with u_2 in P_1 . The resulting graph G is shown in Fig. 2.7.

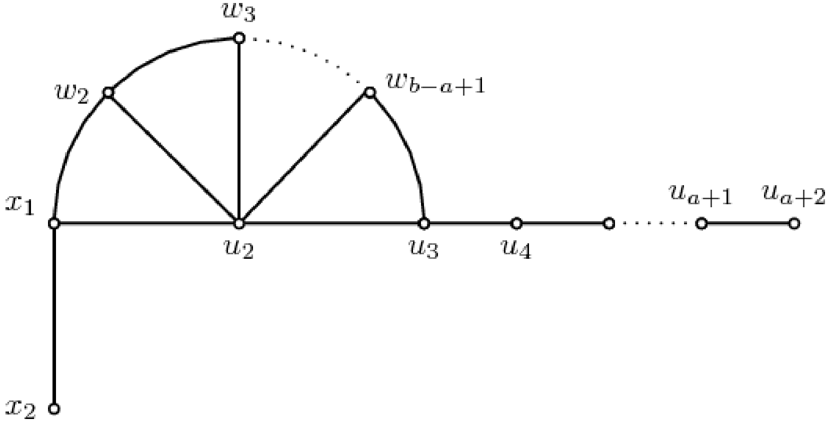

Fig 2.7

It is easy to verify that

$$e_3(x_1x_2) = a$$

$$e_{D_3}(x_1x_2) = b$$

$$e_3(u_iu_{i+1}) = \begin{cases} a-i, & \text{if } 1 \le i \le \left\lfloor \frac{a}{2} \right\rfloor \\ i-1, & \text{if } \left\lfloor \frac{a}{2} \right\rfloor < i \le a+1 \end{cases}$$

$$e_{D_3}(u_iu_{i+1}) = \begin{cases} b-1, & \text{if } i=1 \\ b-a+i-1, & \text{if } 2 \le i \le a \text{ for } b-a+i-1 \ge a-i \\ a-i, & \text{if } 2 \le i \le a \text{ for } b-a+i-1 \le a-i \end{cases}$$

$$e_3(u_2w_i) = a-1 \text{ if } 2 \le i \le b-a+1$$

$$e_{D_3}(u_2w_i) = \begin{cases} b-i, & \text{if } 1 \le i \le b-a+1 \text{ for } b-i \ge i \\ i-1, & \text{if } 1 \le i \le b-a+1 \text{ for } b-i \le i \end{cases}$$

$$e_3(w_iw_{i+1}) = \begin{cases} a, & \text{if } 1 \le i \le b-a-1 \\ a-1, & \text{if } i = b-a \\ a-2, & \text{if } i = b-a+1 \text{ for } a-2 \ge a \\ a-1, & \text{if } i = b-a+1 \text{ for } a-2 \le a-1 \end{cases}$$

$$e_{D_3}(w_iw_{i+1}) = \begin{cases} b-1, & \text{if } 1 \le i \le b-a \\ b-a+1, & \text{if } i = b-a+1 \text{ for } b-a+2 \ge a-2 \\ a-2, & \text{if } i = b-a+1 \text{ for } b-a+2 \le a-2 \end{cases}$$

It is easy to verify that there is no edge e in G with $e_3(e) > a$ and $e_{D3}(e) > b$. Thus $d_3 = a$ and $D_3 = b$ as $a < b$.

Problem 2.16. Characterize the graphs such that $C_{D3}(G) = C_3(G)$

Problem 2.17. Characterize the graphs such that $P_{D3}(G) = P_3(G)$

Problem 2.18. Characterize the graphs such that $C_{D3}(G) = P_{D3}(G)$

Problem 2.19. Is every graph an edge-to-edge detour center of some graph?

Remark 2.20. The edge-to-edge detour center of every connected graph does not lie in a single block of G. For the Path P_{2n+1} of order 2n + 1, the edge-to-edge detour center is always P_3, which does not lie in a single block.

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